



Rewarding Learning

**ADVANCED
General Certificate of Education
2025**

Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

[AMT11]

THURSDAY 29 MAY, MORNING

**MARK
SCHEME**

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

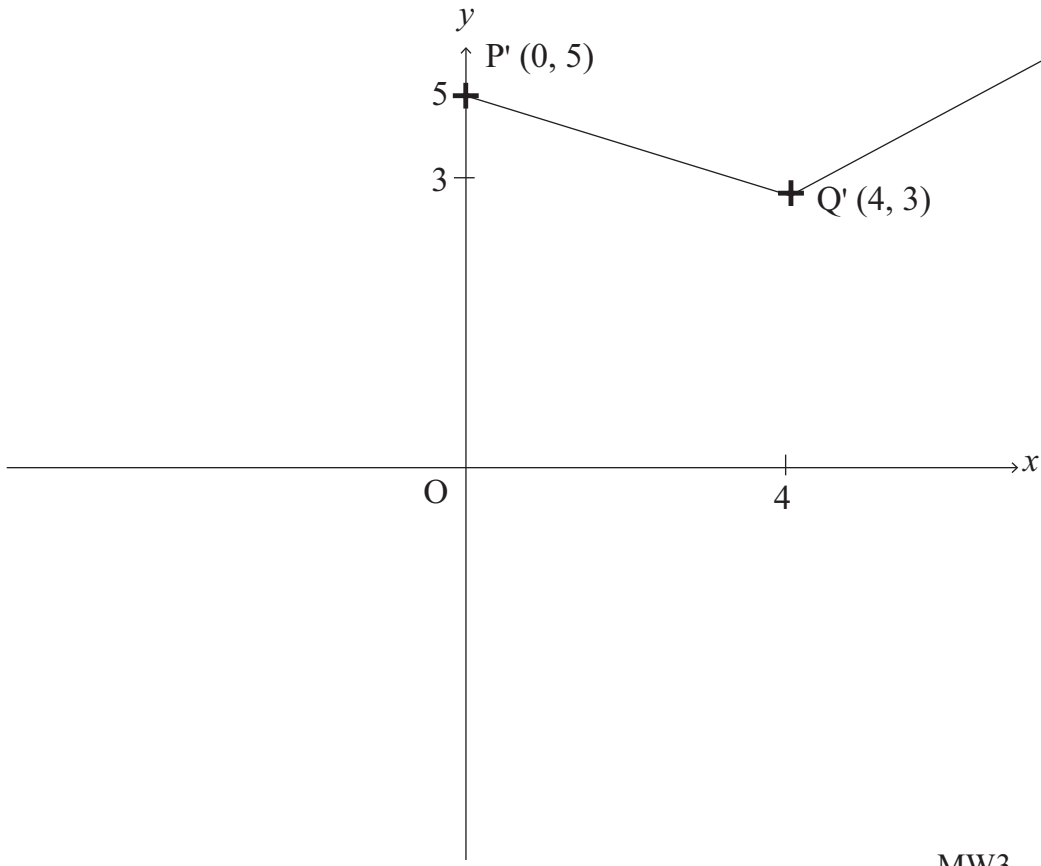
Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

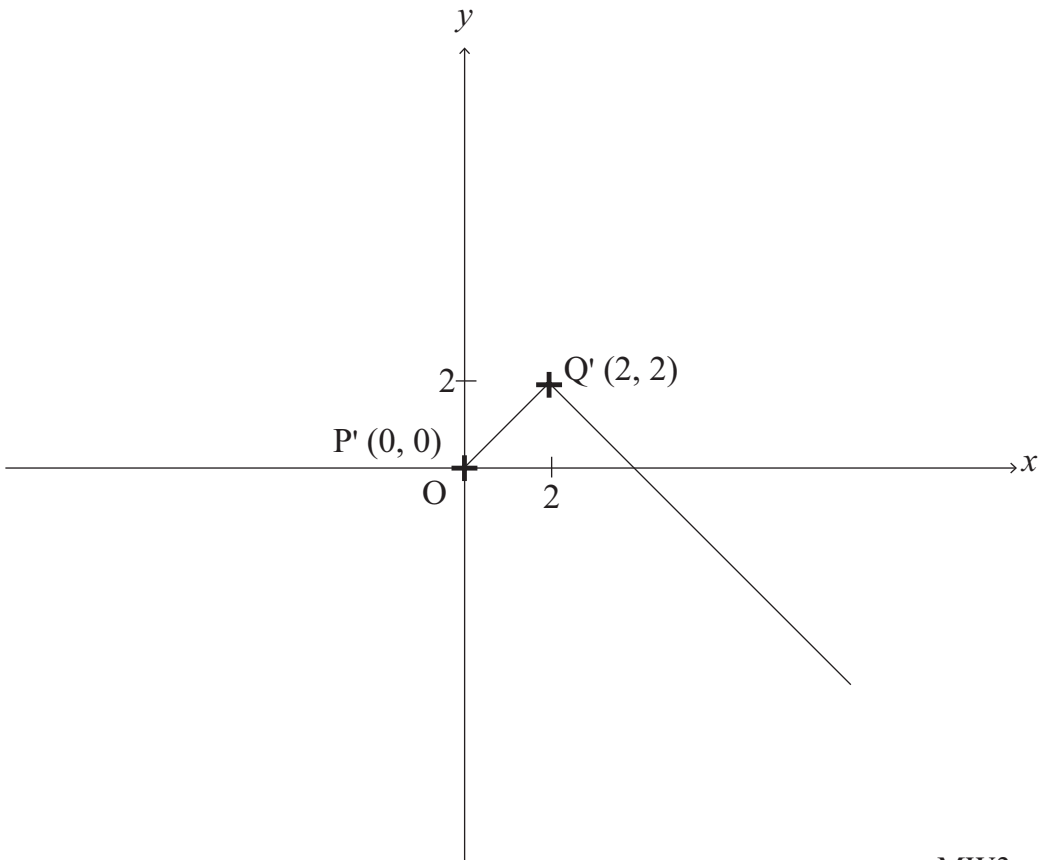
		AVAILABLE MARKS	
1	(i) $\frac{1}{2}, \frac{8}{5}, 3, \frac{32}{7}$ Diverges	MW1 MW1	4
	(ii) $-\frac{1}{3}, -\frac{1}{2}, -\frac{5}{9}, -\frac{7}{12}$ Converges	MW1 MW1	
2	(i) $u_4 = a + 3d = 89$	M1	9
	$u_9 = a + 8d = 69$	MW1	
	$5d = -20$	M1	
	$d = -4$	W1	
	$a + 3(-4) = 89$ $a = 101$	MW1	
(ii) u_n is negative			
$\therefore u_n = a + (n-1)d < 0$			
$101 - 4(n-1) < 0$	M1 MW1		
$105 - 4n < 0$			
$n > 26.25$	W1		
$\therefore 27$ terms	W1		

3 (i)



MW3

(ii)



MW3

AVAILABLE
MARKS

6

4 (i) Arc length = $r\theta$

$$14\pi = 10 \times \theta$$

M1

$$\theta = \frac{7\pi}{5}$$

W1

(ii) Major sector area = $\frac{1}{2}r^2\theta$

$$= \frac{1}{2} \times 10^2 \times \frac{7\pi}{5}$$

M1W1

$$= 70\pi$$

$$\text{Area of circle} = \pi r^2 = 100\pi$$

MW1

$$\therefore \text{Wasted area} = 100\pi - 70\pi$$

$$= 30\pi \text{ cm}^2 (94.2 \text{ cm}^2)$$

MW1

Alternative Solution

$$\text{Angle of minor sector } \theta' = 2\pi - \frac{7\pi}{5} = \frac{3\pi}{5}$$

M1W1

$$\text{Minor sector area} = \frac{1}{2}r^2\theta'$$

$$= \frac{1}{2} \times 10^2 \times \frac{3\pi}{5}$$

M1

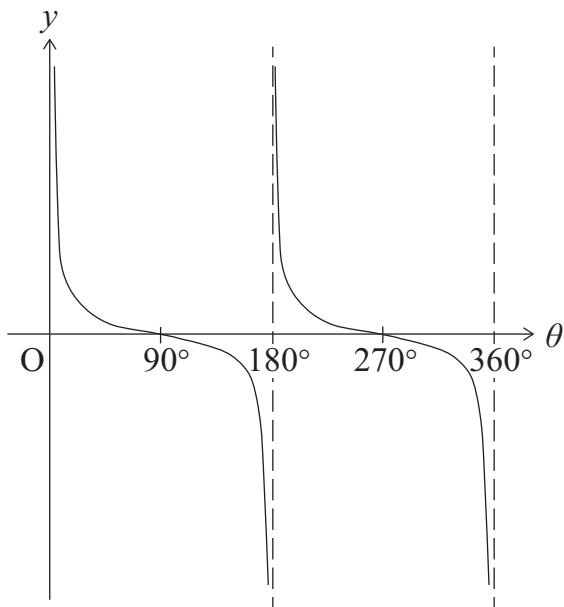
$$= 30\pi \text{ cm}^2 (94.2 \text{ cm}^2)$$

W1

AVAILABLE
MARKS

6

5 (a)



MW3

(b) (i) LHS = $2 \cot 2\theta$

M1

$$= \frac{2}{\tan 2\theta}$$

MW1

$$= \frac{2}{\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)}$$

MW1

$$= \frac{1 - \tan^2 \theta}{\tan \theta}$$

$$= \frac{1}{\tan \theta} - \tan \theta$$

MW1

$$= \cot \theta - \tan \theta = \text{RHS} \quad \text{as required}$$

Alternative Solution

$$\text{RHS} = \cot \theta - \tan \theta$$

M1

$$= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$$

MW1

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{2(\cos^2 \theta - \sin^2 \theta)}{2 \sin \theta \cos \theta}$$

MW1

$$= \frac{2 \cos 2\theta}{\sin 2\theta}$$

MW1

$$= 2 \cot 2\theta = \text{LHS} \quad \text{as required}$$

AVAILABLE
MARKS

(ii) $\cot \theta - \tan \theta = -2$

$\therefore 2 \cot 2\theta = -2$

$\tan 2\theta = -1$

$\Rightarrow 2\theta = 135^\circ, 315^\circ, 495^\circ, 675^\circ$

$\Rightarrow \theta = 67.5^\circ, 157.5^\circ, 247.5^\circ, 337.5^\circ$

or $\theta = 67.5^\circ, 158^\circ, 248^\circ, 338^\circ$ (3 sf)

Alternative Solution

$\frac{1}{\tan \theta} - \tan \theta = -2$

$\tan^2 \theta - 2 \tan \theta - 1 = 0$

$\tan \theta = 1 + \sqrt{2}$

$\theta = 67.5^\circ, 247.5^\circ$

or $\theta = 67.5^\circ, 248^\circ$

$\tan \theta = 1 - \sqrt{2}$

$\theta = 157.5^\circ, 337.5^\circ$

$\theta = 158^\circ, 338^\circ$ (3 sf)

AVAILABLE
MARKS

M1

MW1

MW2

MW1

M1

MW1

MW1

MW2

12

6 (a) $f'(x) = 2e^{2x+1} \operatorname{cosec} 3x + e^{2x+1}(-\operatorname{cosec} 3x \cot 3x)(3)$
 $= 2e^{2x+1} \operatorname{cosec} 3x - 3e^{2x+1} \operatorname{cosec} 3x \cot 3x$

M1 MW2
W1

Alternative Solution

$$f(x) = \frac{e^{2x+1}}{\sin 3x}$$

$$f'(x) = \frac{2e^{2x+1} \sin 3x - e^{2x+1} (\cos 3x)(3)}{\sin^2 3x}$$

$$= 2e^{2x+1} \operatorname{cosec} 3x - 3e^{2x+1} \operatorname{cosec} 3x \cot 3x$$

M1 MW2

W1

(b) $2 + \left(\frac{10xy^3 - 15x^2y^2 \frac{dy}{dx}}{y^6} \right) = 0$

M2 MW3

When $x = -1, y = 1$

$$2 + \left(\frac{-10 - 15 \frac{dy}{dx}}{1} \right) = 0$$

M1

$$\therefore \frac{dy}{dx} = -\frac{8}{15}$$

W1

$y - y_1 = m(x - x_1)$ at $(-1, 1)$

$$y - 1 = -\frac{8}{15}(x + 1)$$

M1

$$\therefore 8x + 15y - 7 = 0$$

W1

Alternative Solution

$$2xy^3 + 5x^2 = 3y^3$$

$$\left(2y^3 + 6xy^2 \frac{dy}{dx} \right) + 10x = 9y^2 \frac{dy}{dx}$$

M2 MW3

When $x = -1, y = 1$

$$\left(2 - 6 \frac{dy}{dx} \right) - 10 = 9 \frac{dy}{dx}$$

M1

$$\therefore \frac{dy}{dx} = -\frac{8}{15}$$

W1

$y - y_1 = m(x - x_1)$ at $(-1, 1)$

$$y - 1 = -\frac{8}{15}(x + 1)$$

M1

$$\therefore 8x + 15y - 7 = 0$$

W1

AVAILABLE
MARKS

13

$$\begin{aligned}
 7 \quad (i) \quad \int \sin^2 x \, dx &= \int \frac{1}{2} (1 - \cos 2x) \, dx \\
 &= \frac{1}{2} x - \frac{1}{4} \sin 2x + c \\
 &= \frac{1}{2} x - \frac{1}{4} (2 \sin x \cos x) + c \\
 &= \frac{1}{2} x - \frac{1}{2} \sin x \cos x + c
 \end{aligned}$$

M1 W1

MW2

MW1

$$\begin{aligned}
 (ii) \quad V &= \pi \int_0^{2\pi} (2 - \sin x)^2 \, dx \\
 &= \pi \int_0^{2\pi} (4 - 4 \sin x + \sin^2 x) \, dx \\
 &= \pi \left[4x + 4 \cos x + \frac{1}{2} x - \frac{1}{2} \sin x \cos x \right]_0^{2\pi} \\
 &= \pi [(8\pi + 4 + \pi - 0) - (0 + 4 + 0 - 0)] \\
 &= \pi [9\pi]
 \end{aligned}$$

M2 MW1

MW1

MW2

$$V = 9\pi^2 \text{ cubic units}$$

W1

12

AVAILABLE
MARKS

8 (i) $f'(x) = 6(1 + \ln x) \times \frac{1}{x}$ M1 MW1

Stationary point

$\Rightarrow \frac{6}{x} (1 + \ln x) = 0$ M1

$\frac{6}{x} \neq 0 \therefore 1 + \ln x = 0$

$\therefore x = e^{-1} = \frac{1}{e}$ W1

$f''(x) = -\frac{6}{x^2} (1 + \ln x) + \frac{6}{x^2}$ M2 W1

$= -\frac{6}{x^2} \ln x$

When $x = \frac{1}{e} : f''(x) = -6e^2 \ln(e^{-1}) = 6e^2$ M1

$\therefore f''(x) > 0$

\therefore minimum turning point MW1

(ii) $x = 0$ MW1

(iii) Domain: $x \in \mathbb{R}, x > 0$ MW1

Range: $f(x) \in \mathbb{R}, f(x) \geq 0$ MW1

(iv) $h(x) = 3(2 + x)^2 \quad x \in \mathbb{R}$ MW1

(v) Let $y = \ln x - 1$ M1

$\ln x = y + 1$

$x = e^{y+1}$ W1

$\therefore g^{-1}(x) = e^{x+1}, \text{ domain } x \in \mathbb{R}$ MW2

AVAILABLE MARKS

17

9 (i)

x	$\frac{2}{5 + \sqrt{x}}$
0	$\frac{2}{5} = 0.4$
0.25	$\frac{4}{11} = 0.3636\dots$
0.5	0.35044...
0.75	0.3409...
1	$\frac{1}{3} = 0.333\dots$

MW1

MW1

MW1

$$\int_0^1 \frac{2}{5 + \sqrt{x}} dx \approx \frac{1}{2} (0.25) \left[0.4 + \frac{1}{3} + 2(0.3636\dots + 0.35044\dots + 0.3409\dots) \right] \text{M1}$$

$$\approx 0.35542\dots$$

$$\approx 0.355 \text{ (3 sf)}$$

W1

(ii) $u = 5 + \sqrt{x}$

$$\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

M1 W1

$$2\sqrt{x} du = dx$$

$$\therefore dx = 2(u - 5) du$$

$$\text{When } x = 0 \Rightarrow u = 5$$

$$\text{When } x = 1 \Rightarrow u = 6$$

MW1

$$\int_0^1 \frac{2}{5 + \sqrt{x}} dx = \int_5^6 \frac{2}{5 + u - 5} \times 2(u - 5) du$$

M1 MW2

$$= \int_5^6 \frac{4}{u} (u - 5) du$$

$$= \int_5^6 4 - \frac{20}{u} du$$

$$= [4u - 20 \ln u]_5^6$$

MW1

$$= [24 - 20 \ln(6)] - [20 - 20 \ln(5)]$$

$$= 4 + 20 \ln\left(\frac{5}{6}\right) = 0.354 \text{ (3 sf)}$$

MW1

(iii) Use more strips / ordinates (or smaller intervals) to improve the approximation.

MW1

AVAILABLE
MARKS

14

10 (a) (i)	$2\sqrt{2} \cos x + \sin x = R (\cos x \cos \alpha + \sin x \sin \alpha)$	MW1
	$\Rightarrow R \cos \alpha = 2\sqrt{2} \quad \text{and} \quad R \sin \alpha = 1$	MW1
	$\Rightarrow \tan \alpha = \frac{1}{2\sqrt{2}}$	M1
	$\alpha = 19.5^\circ$	W1
	$\Rightarrow R = \sqrt{(2\sqrt{2})^2 + 1^2}$	M1
	$R = 3$	W1
(ii)	$3 + 4\sqrt{2} \cos x + 2 \sin x$ can be rewritten as	
	$3 + 2(3\cos(x - 19.5^\circ))$	M1
	Minimum occurs when $\cos(x - 19.5^\circ) = -1$	M1
	\therefore Minimum value = $3 + 6(-1)$	
	$= -3$	W1

AVAILABLE MARKS

(b) (i) $x = 3 \cos t + 4 \sin t + 2$

$$\frac{dx}{dt} = -3 \sin t + 4 \cos t$$

M1 W1

$$y = 4 \cos t - 3 \sin t - 1$$

$$\frac{dy}{dt} = -4 \sin t - 3 \cos t$$

W1

$$\Rightarrow \frac{dy}{dx} = (-4 \sin t - 3 \cos t) \times \left(\frac{1}{-3 \sin t + 4 \cos t} \right)$$

M1

$$= (2 - x) \times \left(\frac{1}{y + 1} \right)$$

MW1

$$\therefore \frac{dy}{dx} = \frac{2 - x}{y + 1} \text{ as required}$$

(ii) $\frac{dy}{dx} = \frac{2 - x}{y + 1}$

$$\int (y + 1) dy = \int (2 - x) dx$$

M1 W1

$$\frac{1}{2} y^2 + y = 2x - \frac{1}{2} x^2 + c$$

MW2

When $x = 5$, $y = 3$

$$\Rightarrow \frac{9}{2} + 3 = 10 - \frac{25}{2} + c$$

M1

$$\therefore c = 10$$

W1

$$\therefore \frac{1}{2} y^2 + y = 2x - \frac{1}{2} x^2 + 10$$

$$x^2 + y^2 - 4x + 2y - 20 = 0$$

MW1

AVAILABLE
MARKS

21

11 (i)

$$2x^2 + 7x + 3 \sqrt{\frac{2}{4x^2 + 11x + 7}} - \frac{4x^2 + 14x + 6}{-3x + 1}$$

M1 MW1

W1

$$\frac{-3x + 1}{(1 + 2x)(3 + x)} \equiv \frac{A}{1 + 2x} + \frac{B}{3 + x}$$

M1

$$-3x + 1 \equiv A(3 + x) + B(1 + 2x)$$

M1

$$\text{When } x = -3 \Rightarrow 10 = 0 - 5B$$

M1

$$B = -2$$

W1

$$\text{When } x = -\frac{1}{2} \Rightarrow \frac{5}{2} = \frac{5}{2}A + 0$$

$$A = 1$$

MW1

$$\therefore \frac{4x^2 + 11x + 7}{(1 + 2x)(3 + x)} \equiv 2 + \frac{1}{1 + 2x} - \frac{2}{3 + x}$$

MW1

Alternative Solution

$$\frac{4x^2 + 11x + 7}{(1 + 2x)(3 + x)} \equiv A + \frac{B}{1 + 2x} + \frac{C}{3 + x}$$

M1

$$4x^2 + 11x + 7 \equiv A(1 + 2x)(3 + x) + B(3 + x) + C(1 + 2x)$$

M1 W1

$$\text{When } x = -3 \Rightarrow 10 = 0 + 0 - 5C$$

M1

$$C = -2$$

W1

$$\text{When } x = -\frac{1}{2} \Rightarrow \frac{5}{2} = 0 + \frac{5}{2}B + 0$$

$$B = 1$$

MW1

$$\text{When } x = 0 \Rightarrow 7 = 3A + 3B + C$$

M1

$$7 = 3A + 3 - 2$$

$$A = 2$$

W1

$$\therefore \frac{4x^2 + 11x + 7}{(1 + 2x)(3 + x)} \equiv 2 + \frac{1}{1 + 2x} - \frac{2}{3 + x}$$

MW1

AVAILABLE
MARKS

$$(ii) \frac{4x^2 + 11x + 7}{(1 + 2x)(3 + x)} \equiv 2 + \frac{1}{1 + 2x} - \frac{2}{3 + x} \quad \text{M1}$$

$$\frac{1}{1 + 2x} = (1 + 2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2!} (2x)^2 + \dots$$

$$= 1 - 2x + 4x^2 + \dots \quad \text{M1 W1}$$

$$\frac{2}{3 + x} = 2(3 + x)^{-1} = 2(3^{-1}) \left(1 + \frac{x}{3}\right)^{-1} \quad \text{MW1}$$

$$= \frac{2}{3} \left(1 + (-1)\left(\frac{x}{3}\right) + \frac{(-1)(-2)}{2!} \left(\frac{x}{3}\right)^2 + \dots\right) \quad \text{M1 W1}$$

$$= \frac{2}{3} \left(1 - \frac{x}{3} + \frac{x^2}{9} + \dots\right)$$

Hence expansion gives

$$2 + (1 - 2x + 4x^2 + \dots) - \frac{2}{3} \left(1 - \frac{x}{3} + \frac{x^2}{9} + \dots\right) \quad \text{M1}$$

$$= \frac{7}{3} - \frac{16}{9}x + \frac{106}{27}x^2 \quad \text{W1}$$

AVAILABLE
MARKS

17

- 12 (i) Let $f(x) = \ln x - x + 2 = 0$ MW1
 $f(3) = 0.09861 \dots$ MW1
 $f(4) = -0.61370 \dots$ MW1

Since there is a change of sign in the value of $f(x)$ between $x = 3$ and $x = 4$, and $f(x)$ is continuous in this range, then the x coordinate of B lies between $x = 3$ and $x = 4$ M1

(ii) $f(x) = \ln x - x + 2$ M1

$f'(x) = \frac{1}{x} - 1$ M1 W1

$x_1 = 3 - \frac{\ln(3) - 3 + 2}{\frac{1}{3} - 1} = 3.14791 \dots$ M1

$x_1 = 3.148$ (3 dp) MW1

(iii) Area = $\int_1^{3.148} (\ln x - x + 2) dx$ MW1

Integration by parts for $\ln x$

$u = \ln x \quad \frac{dv}{dx} = 1$ M2

$\frac{du}{dx} = \frac{1}{x} \quad v = x$ W2

$\therefore \text{Area} = \left([x \ln x]_1^{3.148} - \int_1^{3.148} 1 dx \right) + \left[-\frac{x^2}{2} + 2x \right]_1^{3.148}$ MW3

$= \left[x \ln x - x - \frac{x^2}{2} + 2x \right]_1^{3.148}$

$= \left[x \ln x - \frac{x^2}{2} + x \right]_1^{3.148}$

$= \left[3.148 \ln(3.148) - \frac{3.148^2}{2} + 3.148 \right] - \left[0 - \frac{1}{2} + 1 \right]$ M1

$= 1.30307 \dots$

$= 1.30 \text{ units}^2$ (3 sf) W1

Total

AVAILABLE MARKS

19

150